The Derivative

2.1 المشتقة

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$$\frac{df(x)}{dx}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Examples: find the derivative of following functions

$$f(x)=2x+1$$
$$f(x)=x^2$$
$$f(x)=\sqrt{x}$$

Solution(1)_

$$\frac{df(x)}{dx}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$
$$f(x)=2x+1$$
$$f(x+\Delta x)=2(x+\Delta x)+1$$
$$\frac{df(x)}{dx}=\lim_{\Delta x\to 0}\frac{(2x+2\Delta x+1)-(2x+1)}{\Delta x}=\lim_{\Delta x\to 0}\frac{2x+2\Delta x+1-2x-1}{\Delta x}$$
$$\frac{df(x)}{dx}=\lim_{\Delta x\to 0}\frac{2\Delta x}{\Delta x}=\lim_{\Delta x\to 0}2=2$$

Solution(2)_

$$\frac{df(x)}{dx}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$
$$f(x)=x^2$$
$$f(x+\Delta x)=(x+\Delta x)^2=x^2+2x\Delta x+\Delta x^2$$
$$\frac{df(x)}{dx}=\lim_{\Delta x\to 0}\frac{x^2+2x\Delta x+\Delta x^2-x^2}{\Delta x}=\lim_{\Delta x\to 0}\frac{2x\Delta x-\Delta x^2}{\Delta x}$$
$$\frac{df(x)}{dx}=\lim_{\Delta x\to 0}\frac{\Delta x(2x+\Delta x)}{\Delta x}=\lim_{\Delta x\to 0}2x+\Delta x=2x+0=2x$$

Solution(3)

$$\frac{df(x)}{dx}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$f(x)=\sqrt{x}$$

$$f(x+\Delta x)=\sqrt{(x+\Delta x)}$$

$$\frac{df(x)}{dx}=\lim_{\Delta x\to 0}\frac{\sqrt{(x+\Delta x)}-\sqrt{x}}{\Delta x}=\lim_{\Delta x\to 0}\frac{\sqrt{(x+\Delta x)}-\sqrt{x}}{\Delta x}\times\frac{\sqrt{(x+\Delta x)}+\sqrt{x}}{\sqrt{(x+\Delta x)}+\sqrt{x}}$$

$$=\lim_{\Delta x\to 0}\frac{\left(\sqrt{(x+\Delta x)}-\sqrt{x}\right)\left(\sqrt{(x+\Delta x)}+\sqrt{x}\right)}{\Delta x\left(\sqrt{(x+\Delta x)}+\sqrt{x}\right)}$$

$$\because (z-y)(z+y)=(z^2-y)$$

$$\therefore \left(\sqrt{(x+\Delta x)}-\sqrt{x}\right)\left(\sqrt{(x+\Delta x)}+\sqrt{x}\right)=\left(\sqrt{(x+\Delta x)}\right)^2-\left(\sqrt{x}\right)^2=(x+\Delta x)-x=\Delta x$$

$$\frac{df(x)}{dx}=\lim_{\Delta x\to 0}\frac{\Delta x}{\Delta x\left(\sqrt{(x+\Delta x)}+\sqrt{x}\right)}=\lim_{\Delta x\to 0}\frac{1}{\left(\sqrt{(x+\Delta x)}+\sqrt{x}\right)}=\frac{1}{\sqrt{(x+0)}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$$

$$\frac{df(x)}{dx}=\frac{1}{2\sqrt{x}}$$

The properties of derivative

$$\frac{d}{dx}(cf(x))=c\frac{d}{dx}(f(x)) \quad \text{where c is constant}$$

$$\frac{d}{dx}(x^n)=nx^{n-1}$$

$$\frac{d}{dx}(x^n+c)^m=m(x^n+c)^{m-1}\times\frac{d}{dx}(x^n+c)$$

$$\frac{d}{dx}(f(x)\times g(x))=\left(f(x)\times\frac{d}{dx}g(x)\right)+\left(g(x)\times\frac{d}{dx}f(x)\right)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\left(\frac{\left(g(x)\times\frac{d}{dx}f(x)\right)-f(x)\times\frac{d}{dx}g(x)}{g(x)^2}\right)$$

Example: Show that $\frac{d}{dx}(cf(x))=c\frac{d}{dx}(f(x)) \quad \text{where c is constant}$

$$\frac{dG(x)}{dx}=\lim_{\Delta x\to 0}\frac{G(x+\Delta x)-G(x)}{\Delta x}$$

$$G(x)=cf(x),\quad G(x)=cf(x+\Delta x)$$

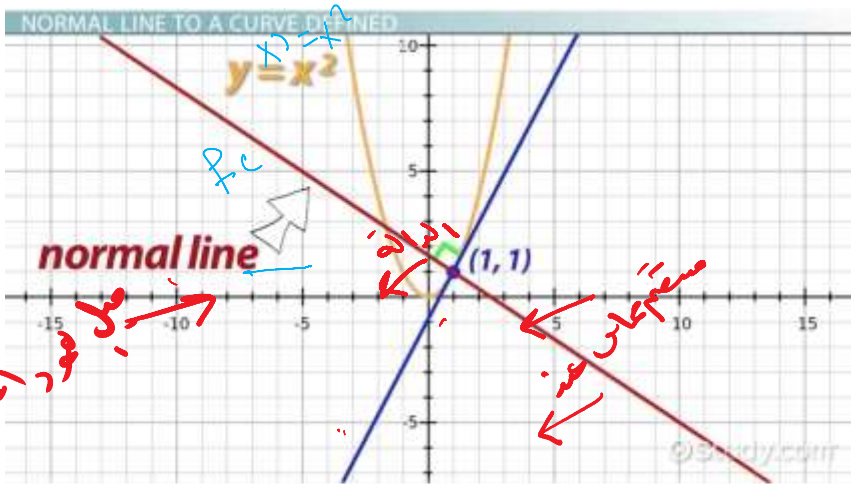
$$\frac{dG(x)}{dx}=\lim_{\Delta x\to 0}\frac{cf(x+\Delta x)-cf(x)}{\Delta x}=c\times\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}=c\frac{df(x)}{dx}$$

2.2 الميل

ميل المستقيم العمود وميل المستقيم المماس

المستقيم العمود و
المستقيم المماس
بالشكل الاتي

ان ميل
وميل
موضح



ان الصيغه الرياضيه لميل المستقيم المماس هي

(x_0, y_0)

$$\frac{y-y_0}{x-x_0}=\frac{d}{dx}f(x)\Big|_{x=x_0}$$

ان الصيغه الرياضيه لميل المستقيم العمود هي

$$\frac{y-y_0}{x-x_0}=\frac{-1}{\frac{d}{dx}f(x)\Big|_{x=x_0}}$$

س1: احسب ميل المستقيم العمود وميل المستقيم المماس للداله   f(x)=2x²-2x+5 عند النقطه (1,3)

ان ميل المستقيم المماس يعطى بالعلاقه

$$\frac{y-y_0}{x-x_0}=\frac{d}{dx}f(x)\Big|_{x=x_0} \qquad x_0=1, \qquad y_0=3$$

$$f(x)=2x^2-2x+5$$

(x_0, y_0)

$(1, 3)$

$x = x_0 = 1$

$$\frac{d}{dx}f(x)\Big| = \frac{d}{dx}\left(2x^2 - 2x + 5 \quad\right) = 4x - 2$$

$$\frac{d}{dx}f(x)\Big|_{x=x_0} = \left(4x-2\right)\Big|_{x=1} = 4(1) - 2 = 2$$

$$\frac{y-y_0}{x-x_0} = \frac{d}{dx}f(x)\Big|_{x=x_0} \quad \Rightarrow \quad \frac{y-3}{x-1} = 2$$

$$y - 3 = 2\left(x-1\right) \quad \Rightarrow \quad y - 3 = 2x - 2 \quad \Rightarrow y = 2x - 2 + 3$$

$$y = 2x + 1$$

اما ان ميل المستقيم العمود

$$\frac{y-y_0}{x-x_0} = \frac{-1}{\frac{d}{dx}f(x)\Big|_{x=x_0}}$$

ع صباحا

$$\frac{y-3}{x-1} = \frac{-1}{2} \quad \Rightarrow y - 3 = \frac{-1}{2}\left(x-1\right)$$

$$y = \frac{-1}{2}\left(x-1\right) + 3$$

س2: احسب ميل المستقيم العمود وميل المستقيم المماس للداله $f(x)=x^2$ عند النقطه (2,4)

الحل :ان ميل المستقيم المماس يعطى بالعلاقه

$$\frac{y-y_0}{x-x_0} = \frac{d}{dx}f(x)\Big|_{x=x_0} \qquad x_0 = 2, \qquad y_0 = 4$$

$$f(x) = x^2$$

$$\frac{d}{dx}f(x)\Big| = \frac{d}{dx}\left(x^2\right) = 2x$$

$$\frac{d}{dx}f(x)\Big|_{x=x_0} = \left(2x\right)\Big|_{x=2} = 2(2) = 4$$

$$\frac{y-y_0}{x-x_0} = \frac{d}{dx}f(x)\Big|_{x=x_0} \quad \Rightarrow \quad \frac{y-4}{x-2} = 4$$

$$y - 4 = 4\left(x-2\right) \quad \Rightarrow \quad y - 4 = 4x - 8 \quad \Rightarrow y = 4x - 8 + 4$$

$$y = 4x - 4$$

اما ان ميل المستقيم العمود

$$\frac{y-y_0}{x-x_0}=\frac{-1}{\frac{d}{dx}f(x)\Big|_{x=x_0}}$$

$$\frac{y-4}{x-2}=\frac{-1}{4}\Rightarrow y-4=\frac{-1}{4}(x-2)$$

$$y=\frac{-1}{4}(x-2)+4 \quad \Rightarrow y=\frac{-1}{4}x+\frac{1}{2}+4 \quad \Rightarrow \quad y=\frac{-1}{4}x+4.5$$