

Home works

Evaluating Definite Integrals

Evaluate the integrals in Exercises 45-70.

45. $\int_{-1}^1 (3x^2 - 4x + 7) dx$

46. $\int_0^1 (8s^3 - 12s^2 + 5) ds = 8 \frac{s^4}{4} - 12 \frac{s^3}{3} + 5s \Big|_0^1 = 2 - 4 + 5 = 3$

47. $\int_1^2 \frac{4}{v^2} dv = 4 \int_1^2 v^{-2} dv = 4 \left[-v^{-1} \right]_1^2 = 4 \left[-\frac{1}{2} + 1 \right] = 2$

48. $\int_1^{27} x^{-4/3} dx = \frac{x^{-1/3+1}}{-1/3+1} \Big|_1^{27} = \frac{3}{2} x^{2/3} \Big|_1^{27} = \frac{3}{2} (9 - 1) = 12$

49. $\int_1^4 \frac{dt}{t\sqrt{t}} = \int_1^4 t^{-3/2} dt = \int_1^4 t^{-3/2} dt = -2t^{-1/2} \Big|_1^4 = -2 \left[\frac{1}{2} - 1 \right] = 2$

50. $\int_1^4 \frac{(1+\sqrt{u})^{1/2}}{\sqrt{u}} du$

51. $\int_0^1 \frac{36 dx}{(2x+1)^3}$

52. $\int_0^1 \frac{-5 dr}{\sqrt[3]{(7-5r)^2}}$

53. $\int_{1/8}^1 x^{-1/3} (1-x^{2/3})^{3/2} dx$

54. $\int_0^{1/2} 36x^3 (1+9x^4)^{-3/2} dx$

55. $\int_0^\pi \sin^2 5r dr$

56. $\int_0^{\pi/4} \cos^2 \left(4t - \frac{\pi}{4} \right) dt$

57. $\int_0^{\pi/3} \sec^2 \theta d\theta = \tan \theta \Big|_0^{\pi/3} = \tan \frac{\pi}{3} = \sqrt{3}$

58. $\int_{\pi/4}^{3\pi/4} \csc^2 x dx = -\cot x \Big|_{\pi/4}^{3\pi/4} = -\cot \frac{3\pi}{4} + \cot \frac{\pi}{4} = 1 + 1 = 2$

59. $\int_\pi^{3\pi} \cot^2 \frac{x}{6} dx = \tan \frac{x}{6} - x \Big|_\pi^{3\pi} = \tan \frac{\pi}{2} - 3\pi - \left(\tan \frac{\pi}{2} - \pi \right) = -2\pi$

60. $\int_0^\pi \tan^2 \frac{\theta}{3} d\theta = \int_0^\pi (\sec^2 \frac{\theta}{3} - 1) d\theta = 3 \tan \frac{\theta}{3} - \theta \Big|_0^\pi = 3 \tan \frac{\pi}{3} - \pi = 3\sqrt{3} - \pi$

61. $\int_{-\pi/3}^0 \sec x \tan x dx = \sec x \Big|_{-\pi/3}^0 = 1 - \frac{2}{\sqrt{3}}$

62. $\int_{\pi/4}^{3\pi/4} \csc z \cot z dz = \ln |\csc z| \Big|_{\pi/4}^{3\pi/4} = \ln \sqrt{2} - \ln \sqrt{2} = 0$

63. $\int_0^{\pi/2} 5(\sin x)^{3/2} \cos x dx$

64. $\int_{-1}^1 (-2x) \sin(1-x^2) dx$

65. $\int_{-\pi/2}^{\pi/2} 15 \sin^2 3x \cos 3x dx$

66. $\int_0^{2\pi/3} \cos^{-4} \left(\frac{x}{2} \right) \sin \left(\frac{x}{2} \right) dx$

67. $\int_0^{\pi/2} \frac{2x^3 \sin x \cos x}{\sqrt{1+3\sin^2 x}} dx$

68. $\int_0^{\pi/4} \frac{\sec^2 x}{(1+7\tan x)^{2/3}} dx$

69. $\int_0^{\pi/3} \frac{\tan \theta}{\sqrt{2 \sec \theta}} d\theta$

70. $\int_{\pi^2/36}^{\pi/4} \frac{\cos \sqrt{t}}{\sqrt{t \sin \sqrt{t}}} dt$

Handwritten notes on the left side of the page:

- $\int_0^1 (3x^2 + 7 - 4x) dx = \left[x^3 - 2x^2 + 7x \right]_0^1 = 1 - 2 + 7 = 6$
- $\int_1^2 \frac{4}{v^2} dv = 4 \left[-\frac{1}{v} \right]_1^2 = 4 \left[-\frac{1}{2} + 1 \right] = 2$
- $\int_1^4 \frac{dt}{t\sqrt{t}} = \int_1^4 t^{-3/2} dt = -2t^{-1/2} \Big|_1^4 = -2 \left[\frac{1}{2} - 1 \right] = 2$
- $\int_0^1 \frac{36 dx}{(2x+1)^3} = \int_1^3 \frac{18 du}{u^3} = 18 \left[-\frac{1}{2u^2} \right]_1^3 = 18 \left[-\frac{1}{18} + \frac{1}{2} \right] = 16$
- $\int_0^{\pi/2} 5(\sin x)^{3/2} \cos x dx = \frac{5}{5/2} (\sin x)^{5/2} \Big|_0^{\pi/2} = \frac{2}{5} (1)^{5/2} = \frac{2}{5}$
- $\int_{-\pi/2}^{\pi/2} 15 \sin^2 3x \cos 3x dx = \frac{15}{3} \int_{-\pi/2}^{\pi/2} \sin^2 u du = 5 \int_{-\pi/2}^{\pi/2} \frac{1 - \cos 2u}{2} du = \frac{5}{2} \left[u - \frac{\sin 2u}{2} \right]_{-\pi/2}^{\pi/2} = \frac{5}{2} \left[\frac{\pi}{2} - \frac{\sin \pi}{2} - \left(-\frac{\pi}{2} - \frac{\sin(-\pi)}{2} \right) \right] = 5\pi$

$0 + 3 \cdot 2 \sin x \cos x = 6 \sin x \cos x$

Handwritten notes on the right side of the page:

- $\frac{3x^3}{3} - 4 \frac{x^2}{2} + 7x \Big|_0^1 = 1 - 2 + 7 = 6$
- $\int_0^1 (8s^3 - 12s^2 + 5) ds = 8 \frac{s^4}{4} - 12 \frac{s^3}{3} + 5s \Big|_0^1 = 2 - 4 + 5 = 3$
- $\int_1^2 \frac{4}{v^2} dv = 4 \left[-\frac{1}{v} \right]_1^2 = 4 \left[-\frac{1}{2} + 1 \right] = 2$
- $\int_1^4 \frac{dt}{t\sqrt{t}} = \int_1^4 t^{-3/2} dt = -2t^{-1/2} \Big|_1^4 = -2 \left[\frac{1}{2} - 1 \right] = 2$
- $\int_0^1 \frac{36 dx}{(2x+1)^3} = \int_1^3 \frac{18 du}{u^3} = 18 \left[-\frac{1}{2u^2} \right]_1^3 = 18 \left[-\frac{1}{18} + \frac{1}{2} \right] = 16$
- $\int_0^{\pi/2} 5(\sin x)^{3/2} \cos x dx = \frac{5}{5/2} (\sin x)^{5/2} \Big|_0^{\pi/2} = \frac{2}{5} (1)^{5/2} = \frac{2}{5}$
- $\int_{-\pi/2}^{\pi/2} 15 \sin^2 3x \cos 3x dx = \frac{15}{3} \int_{-\pi/2}^{\pi/2} \sin^2 u du = 5 \int_{-\pi/2}^{\pi/2} \frac{1 - \cos 2u}{2} du = \frac{5}{2} \left[u - \frac{\sin 2u}{2} \right]_{-\pi/2}^{\pi/2} = 5\pi$

$\frac{1}{2} \int (1 + \tan^2 x)^{3/2} \sec^2 x dx = \frac{1}{2} (1 + 7 \tan x)^{3/2+1}$

$\frac{1}{12} \int \sec^2 \theta \tan \theta d\theta = \int \frac{\sec^2 \theta}{\sec \theta} [\sec \tan] d\theta = \int \sec \theta (\sec \theta \tan \theta) d\theta = \frac{1}{2} \sec^2 \theta + C = \frac{1}{2} \frac{1}{\cos^2 \theta} + C$