$\log_2 4 = \log_2 2^2 = 2\log_2 2 = 2\times 1 = 2$

Properties for Expanding Logarithms

Property 1: $\log_a 1 = 0$ – Zero-Exponent Rule

$\log_{10} 1 = 0$, $\log_2(1) = 0$

Property 2: $\log_a a = 1$

$\therefore \log_{10} 10 = 1$, $\log_4 4 = 1$

Property 3: $\log_a(xy) = \log_a x + \log_a y$ – Product Rule

$\log_a(x*y) = \log_a(x) + \log_a(y)$

Property 4: $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$ – Quotient Rule

Property 5: $\log_a x^y = y\log_a x$ – Power Rule

$\log_a x^5 = 5\log_a x$,

Example 1: Use the properties of logarithms to expand $\log_3(x^2y^7)$. $= \log_3 x^2 + \log_3 x^7$

$\log_3(x^2y^7) = \log_3 x^2 + \log_3 y^7$ Use property 3 to rewrite the multiplication as addition.

$= 2\log_3 x + 7\log_3 y$ Use Property 5 to move the exponents out front.

Thus, $\log_3(x^2y^7) = 2\log_3 x + 7\log_3 y$.

Example 2: Use the properties of logarithms to expand $\log_8\left(\frac{\sqrt{x}}{y^3}\right)$ $= \log_8 \sqrt{x} - \log_8 y^3 = \log_8 x^{\frac{1}{2}} - \log_8 y^{3}$

$\log_8\left(\frac{\sqrt{x}}{y^3}\right) = \log_8\left(\frac{x^{1/2}}{y^3}\right)$ Rewrite the radical using rational exponents (fractions).

$= \log_8 x^{1/2} - \log_8 y^3$ Use property 4 to rewrite the division as subtraction.

$= \frac{1}{2}\log_8 x - 3\log_8 y$ Use Property 5 to move the exponents out front.

Thus, $\log_8\left(\frac{\sqrt{x}}{y^3}\right) = \frac{1}{2}\log_8 - 3\log_8 y$.

Example 3: Use the properties of logarithms to expand $\log_4\left(\frac{4x}{y^9}\right)$ $= \log_4(4x) - \log_4 y^9$

$= \boxed{\log_4 4} + \log_4 x - 9\log_4 y$

1

$\log_4\left(\frac{4x}{y^9}\right) = \log_4 4 + \log_4 x - \log_4 y^9$ Use properties 3 and 4 to rewrite the multiplication as addition and the division as subtraction. Note that 4x means 4 times x which is why Property 3 has been used to rewrite the logarithm using addition.

$= \log_4 4 + \log_4 x - 9\log_4 y$ Use Property 5 to move the exponents out front.

$= 1 + \log_4 x - 9\log_4 y$ Use Property 2 to simplify the logarithm.

Example 4: Use the properties of logarithms to expand $\log\left(\frac{\sqrt[4]{x^3}\,y^7}{z^8}\right)$.

$\therefore \sqrt[4]{x^3} = \left(x^3\right)^{\frac{1}{4}} = x^{\frac{3}{4}}$

$\log(x^{\frac{3}{4}}\cdot y^7) - \log z^8$

$\log\left(\frac{\sqrt[4]{x^3}\,y^7}{z^8}\right) = \log\left(\frac{x^{3/4}y^7}{z^8}\right)$ Rewrite the radical using rational exponents (fractions).

$= \log x^{\frac{3}{4}} + \log y^7 - \log z^8$

$= \log x^{3/4} + \log y^7 - \log z^8$ Use properties 3 and 4 to rewrite the multiplication as addition and the division as subtraction.

$= \frac{3}{4}\log x + 7\log y - 8\log z$ Use Property 5 to move the exponents out front.

Thus, $\log\left(\frac{\sqrt[4]{x^3}\,y^7}{z^8}\right) = \frac{3}{4}\log + 7\log y - 8\log z.$

Example 5: Use the properties of logarithms to expand $\ln\left(\frac{x^5}{y^2z^7}\right)$.

$\ln\frac{x^5}{y^2z^7} = \ln x^5 - \ln(y^2z^7)$

$= 5\ln x - [2\ln y + 7\ln z]$

$\ln\left(\frac{x^5}{y^2z^7}\right) = \ln x^5 - \ln y^2 - \ln z^7$ Use property 4 to rewrite the division as subtraction. Since both the y and z terms are in the denominator of the fraction, they are each considered to be division and this makes each of them into subtraction when expanded.

$= 5\ln x - 2\ln y - 7\ln z$ Use Property 5 to move the exponents out front.

Thus, $\ln\left(\frac{x^5}{y^2z^7}\right) = 5\ln x - 2\ln y - 7\ln z.$

Practice Problems

Now it is your turn to try a few practice problems on your own. Work on each of the problems below and then click on the link at the end to check your answers.

Problem 1: Use the properties of logarithms to expand $\log_6\left(\frac{\sqrt[5]{x^3}}{y^9}\right)$.

Problem 2: Use the properties of logarithms to expand $\log\left(\frac{10\sqrt{x}}{y^5}\right)$.

Problem 3: Use the properties of logarithms to expand $\log_8\left(\frac{x^{14}y^7}{z^{15}}\right)$.

Problem 4: Use the properties of logarithms to expand $\ln\left(7x^3y^8\right)$.

Problem 5: Use the properties of logarithms to expand $\log_2\left(\frac{1}{x^5y^3}\right)$.

Neutral logarithm

$$\int_1^x \frac{1}{t}\,dt = \ln x - \ln 1 = \ln x - 0$$

Natural logarithm is the logarithm to the base e.

Notation: $\log_e x \rightarrow \ln x$

DEFINITION The Natural Logarithm Function

$$\ln x = \int_1^x \frac{1}{t}\,dt, \qquad x > 0$$

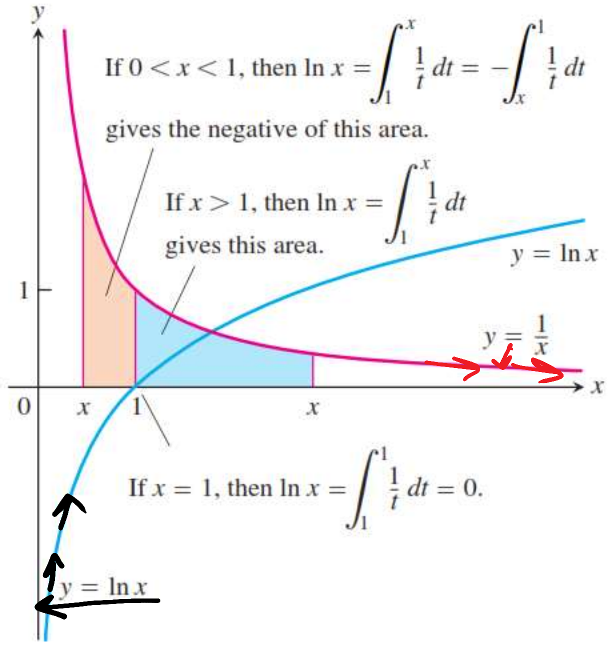

TABLE 7.1 Typical 2-place values of ln *x*

x		$\ln x$
0		undefined
0.05	$\ln 0.05 =$	−3.00
0.5	$\ln 0.5 =$	−0.69
1	$\ln(1) =$	0
2		0.69
3		1.10
4		1.39
10		2.30

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DEFINITION The Number *e*

The number e is that number in the domain of the natural logarithm satisfying

$$\ln(e) = 1$$

$$\frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx}, \qquad u > 0 \qquad (1)$$

$\frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx}$ $\quad \rightarrow \ln 2x = \ln u$

EXAMPLE 1 Derivatives of Natural Logarithms

You Try It

(a) $\frac{d}{dx}\ln 2x = \frac{1}{2x}\frac{d}{dx}(2x) = \frac{1}{\cancel{2x}}(\cancel{2}) = \frac{1}{x}$ $\quad \rightarrow \frac{d}{dx}\ln(2x) = \frac{1}{2x}\frac{d}{dx}(2x)$

You Try It

(b) Equation (1) with $u = x^2 + 3$ gives

$$\frac{d}{dx}\ln(x^2+3) = \frac{1}{x^2+3}\cdot\frac{d}{dx}(x^2+3) = \frac{1}{x^2+3}\cdot 2x = \frac{2x}{x^2+3}$$

$$\frac{d}{dx}\ln(x^2+3) = \frac{1}{(x^2+3)}\frac{d}{dx}(x^2+1) = \frac{1}{x^2+3}(2x)$$

$$\frac{d}{dx}\ln ax = \frac{1}{ax}\cdot\frac{d}{dx}(ax) = \frac{1}{\cancel{ax}}(\cancel{a}) = \frac{1}{x}.$$

$\because \frac{d}{dx}ax = a\frac{d}{dx}x = a\cdot 1 = a$

THEOREM 2 **Properties of Logarithms**

For any numbers $a > 0$ and $x > 0$, the natural logarithm satisfies the following rules:

1. *Product Rule*: $\ln ax = \ln a + \ln x$ $\quad$ ج. $\quad$ لا ضرب ا جمع
2. *Quotient Rule*: قسمة ، طرح $\quad \ln \frac{a}{x} = \ln a - \ln x$
3. *Reciprocal Rule*: $\ln \frac{1}{x} = -\ln x$ $= \ln 1 - \ln x = 0 - \ln x = -\ln x$ $\quad$ Rule 2 with $a = 1$
4. *Power Rule*: $\ln x^r = r\ln x$ $\quad$ r rational

We illustrate how these rules apply.

EXAMPLE 2 Interpreting the Properties of Logarithms

You Try It

(a) $\ln 6 = \ln(2\cdot 3) = \ln 2 + \ln 3$ $\quad$ Product

(b) $\ln 4 - \ln 5 = \ln\frac{4}{5} = \ln 0.8$ $\quad$ Quotient

(c) $\ln\frac{1}{8} = -\ln 8$ $\quad 8 = 2^3$ $\quad$ Reciprocal $= 2^1 \times 2^1 \times 2^1 = 2^3$

$\quad = -\ln 2^3 = -3\ln 2$ $\quad$ Power

EXAMPLE 3 Applying the Properties to Function Formulas

(a) $\ln 4 + \ln\sin x = \ln(4\sin x)$ $\quad$ Product

(b) $\ln\frac{(x+1)}{(2x-3)} = \ln(x+1) - \ln(2x-3)$ $\quad$ Quotient

[Example 5.1] Find the derivative of the following functions.

1- $y = \ln(2x)$

$$y' = \frac{1}{2x}\cdot(2x)' = \frac{\cancel{2}}{\cancel{2}x} = \frac{1}{x}$$

$$\frac{d}{dx}\ln 3x = \frac{1}{3x}\,\frac{d}{dx}3x = \frac{3}{3x} = \frac{1}{x}$$

2- $y = x\ln 3x$

$1 = (x)'$

$$y' = x'\ln 3x + x(\ln 3x)' = \ln 3x + \cancel{x}\cdot\frac{3}{\cancel{3x}} = \ln 3x + 1$$

$$(\ln x)^2 \neq \ln x^2 \neq 2\ln x$$

Find the derivive of the following functions

$(x^n)' = nx^{n-1}$

(1) $\dfrac{d}{dx}(\ln x)^2 = 2\ln x\cdot\dfrac{d}{dx}\ln x = \dfrac{2\ln x}{x}$ →

$$\frac{d}{dx}(\ln x)^2 = 2(\ln(x))^{2-1}\frac{d}{dx}\ln x = 2(\ln x)\left(\frac{1}{x}\right) = \frac{2}{x}\ln x$$

(2) $\dfrac{d}{dx}(x\ln x) = (x)'\cdot\ln x + x\cdot(\ln x)' = \ln x + x\cdot\dfrac{1}{x} = \ln x + 1$

(3) $\boxed{\log_{10}x} = \dfrac{\ln x}{\ln 10}$ → $\log_a x = \dfrac{\ln x}{\ln a}$

القاعدة الاساسية لما نربط بين اللوغارتم الطبيعي . ln و اللوغارتم للاساس a $\log_a$

$$\frac{d}{dx}\log_{10}x = \frac{d}{dx}\frac{\ln x}{\ln 10} = \frac{1}{\ln 10}\frac{d}{dx}\ln x = \frac{1}{(\ln 10)x}$$

$$\log_3 x = \frac{\ln x}{\ln 3}$$

$$\frac{d}{dx}\log_{10}x = \frac{d}{dx}\frac{\ln x}{\boxed{\ln 10}}\ co$$