1.1 Introduction

Probability is the branch of mathematics concerning events and numerical descriptions of how likely they are to occur. The probability of an event is a number between 0 and 1; the larger the probability, the more likely an event is to occur. The higher the probability of an event, the more likely it is that the event will occur. A simple example is the coin. Since the coin is fair, the two outcomes ('heads' and 'tails') are both equally probable; the probability of 'heads' equals the probability of 'tails'; and since no other outcomes are possible, the probability of either 'heads' or 'tails' is 1/2 (which could also be written as 0.5 or 50%). These concepts have been given an axiomatic mathematical formalization in probability theory, which is used widely in areas of study such as statistics, mathematics, science, finance, artificial intelligence, machine learning, computer science, game theory, and philosophy to, for example, draw inferences about the expected frequency of events. Probability theory is also used to describe the underlying mechanics and regularities of complex systems.[3]

1.2 Basic Concept of Probability

What is the Probability of A and B?

The probability of A and B means that we want to know the probability of two events happening at the same time.

P(A and B) = P(A) * P(B).

If the probability of one event doesn't affect the other, you have an **independent** event. All you do is multiply the probability of one by the probability of another.

Examples

Example 1: The odds (probability) of you getting promoted this year are 1/4. The odds of you being audited by the head of department are about 1 in 118. What are the odds that you get promoted and you get audited?

مثال: احتمالات (احتمالية) حصولك على ترقية هذا العام هي 4/1. تبلغ احتمالات أن يتم تدقيقك من قبل رئيس القسم حوالي 1 في 118. ما هي احتمالات حصولك على ترقية وتدقيقك؟

Solution:

Step 1: Multiply the two probabilities together:

p(A and B) = p(A) * p(B) = 1/4 * 1/118 = 0.002.

That's it!

Example 2: The odds of it raining today is 40%; the odds of you getting a goal in one in golf are 0.08%. What are your odds of it raining and you getting a goal in one?

مثال 2: احتمال سقوط المطر اليوم هو 40%، احتمالات حصولك على نقطة تسجيل في لعبة الجولف هي 0.08%. ما هي احتمالات هطول المطر وحصولك على هدف في إحداها؟

Solution:

Step 1: Multiply the probability of A by the probability of B.

P(A and B) = P (A) * P (B) = 0.4 * 0.0008 = 0.00032.

That's it!

Formula for the probability of A and B (dependent events):

$$P(A \text{ and } B) = P(A) * P(B|A)$$

The formula is a little more complicated if your events are dependent, that is if the probability of one event effects another. In order to figure these probabilities out, you must find P (B|A), which is the conditional probability for the event.

تكون الصيغة أكثر تعقيدًا قليلًا إذا كانت الاحداث تعتمد على بعضها البعض، أي إذا كان احتمال وقوع حدث يؤثر على حدث آخر. من أجل معرفة هذه الاحتمالات، يجب عليك إيجاد P (B|A) ، وهو الاحتمال الشرطي للحدث.

Example: You have 52 candidates for a committee. Four are persons aged 18 to 21. If you randomly select one person, and then (without replacing the first person's name), randomly select a second person, what is the probability both people will be between 18 and 21 years old?

مثال: لديك 52 مرشحًا للجنة. أربعة أشخاص تتراوح أعمارهم بين 18 و21 عامًا. إذا اخترت شخصًا واحدًا عشوائيًا، ثم (دون استبدال اسم الشخص الأول)، اخترت عشوائيًا شخصًا ثانيًا، ما احتمال أن يكون عمر كلا الشخصين بين 18 و21 عامًا؟

Solution:

Step 1: Figure out the probability of choosing an 18 to 21 year old on the first draw (drô). As there are 52 possibilities, and 4 are aged 18 to 21, you have a 4/52 = 1/13 chance.

الخطوة 1: معرفة احتمالية اختيار شخص يتراوح عمره بين 18 و21 عامًا في السحب الأول. نظرًا لوجود 52 احتمالًا، و4 منهم تتراوح أعمارهم بين 18 و21 عامًا، فلديك فرصة 4/52 = 1/13.

Step 2: Figure out P(B|A), which is the probability of the next event (choosing a second person aged 18 to 21) given that the first event in Step 1 has already happened.

الخطوة 2: اكتشف P (B|A) ، وهو احتمال الحدث التالي (اختيار شخص ثانٍ يتراوح عمره بين 18 و21 عامًا) نظرًا لأن الحدث الأول في الخطوة 1 قد حدث بالفعل.

There are 51 people left, and only 3 are aged 18 to 21 now, so the probability of choosing a young Person again is 3/51 = 1 / 17.

Step 3: Multiply your probabilities from Step 1(P (A)) and Step 2(P (B|A)) together:

P (A) * P (B|A) = 1/13 * 1/17 = 1/221.

Your odds of choosing two people aged 18 to 21 are 1 out of 221.

2. What is the Probability of A or B?

The probability of A or B depends on if you have mutually exclusive events (ones that cannot happen at the same time) or not.

تعمد احتمال A أو B على ما إذا كان لديك أحداث متنافية (تلك التي لا يمكن أن تحدث في نفس الوقت) أم لا.

If two events A and B are mutually exclusive, the events are called disjoint events. The probability of two disjoint events A or B happening is:

إذا كان الحدثان A وB متنافيان، فإن الحدثين يسمان حدثين منفصلين. احتمال وقوع حدثين منفصلين A أو B هو:

P(A or B) = P (A) + P (B).

Example: What is the probability of choosing one card from a standard deck and getting either a Queen of Hearts or Ace of Hearts? Since you can't get both cards with one draw, add the probabilities:

ما هو احتمال اختيار بطاقة واحدة من المجموعة القياسية والحصول على ملكة القلوب أو آس القلوب؟ بما أنه لا يمكنك الحصول على كلتا البطاقتين بسحب واحد، قم بإضافة الاحتمالات:

P(Queen of Hearts or Ace of Hearts) = P (Queen of Hearts) + P (Ace of Hearts) = 1/52 + 1/52 = 2/52.

If the events A and B are not mutually exclusive, the probability is:

(A or B) = P(A) + P (B) – P (A and B).

Example: What is the probability that a card chosen from a standard deck will be a Jack or a heart? مثال: ما هو احتمال أن تكون البطاقة المختارة من المجموعة القياسية جاك أو قلبًا؟

Solution:

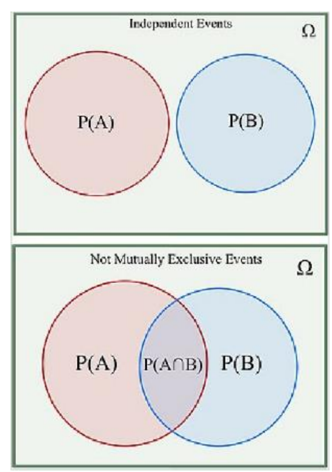
- P (Jack) = 4/52
- P (Heart) = 13/52
- P (Jack of Hearts) = 1/52

So:

P (Jack or Heart) = P (Jack) + P (Heart) – P (Jack of Hearts) = 4/52 + 13/52 – 1/52 = 16/52.

Summary of probabilities

Event	Probability
A	$P(A) \in [0,1]$
not A	$P(A^{\complement}) = 1 - P(A)$
A or B	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A \cup B) = P(A) + P(B)$ if A and B are mutually exclusive
A and B	$P(A \cap B) = P(A\|B)P(B) = P(B\|A)P(A)$ $P(A \cap B) = P(A)P(B)$ if A and B are independent
A given B	$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B\|A)P(A)}{P(B)}$



1.3 Permutations and combinations

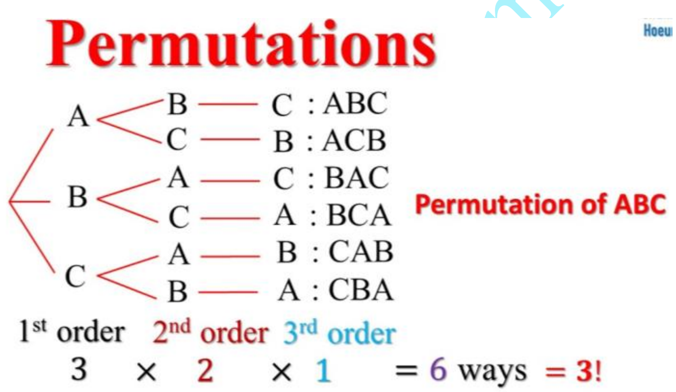
Permutation and Combination are the most fundamental concepts in mathematics that are the ways to arrange a group of objects by selecting them in a specific order and forming their subsets. To arrange groups of data in a specific order permutation and combination formulas are used.

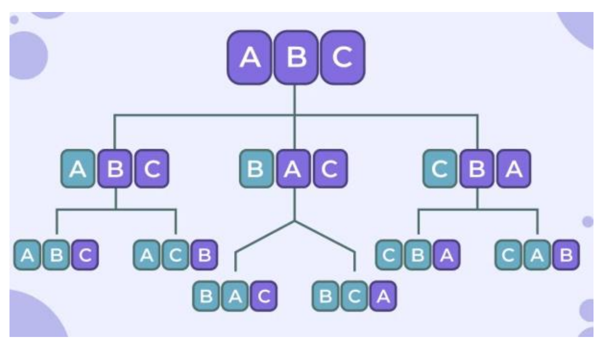
Selecting the data or objects from a certain group is said to be permutation, whereas the order in which they are arranged is called a combination.

ويقال إن اختيار البيانات أو الكائنات من مجموعة معينة هو التباديل، في حين أن الترتيب الذي يتم ترتيبها به يسمى توافيق.

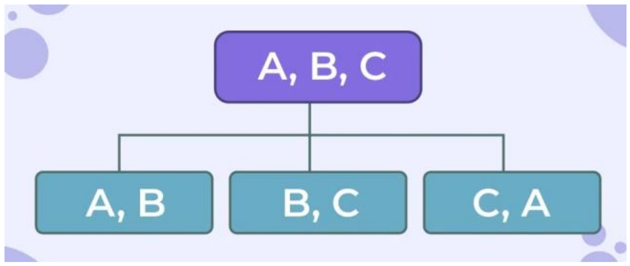
In this article we will study the concept of Permutation and Combination and their formulas, using these to solve many sample problems as well.

For example, if we have two components A,B and C, then the permutation





The combination of any two letters out of three letters A, B, and C is shown below, we notice that in combination the order in which A and B are taken is not important as AB and BA represent the same combination.



Mathematically, the permutation and combination is given by the following equation

Permutations and Combinations

Number of permutations (order matters) of *n* things taken *r* at a time:

$$P(n,r)=\frac{n!}{(n-r)!}$$

Number of combinations (order does not matter) of *n* things taken *r* at a time:

$$C(n,r)=\frac{n!}{(n-r)!r!}$$

Number of different permutations of *n* objects where there are n_1 repeated items, n_2 repeated items, ... n_k repeated items

$$\frac{n!}{n_1!n_2!\ldots n_k!}$$

TABLE I. FOUR BASIC TYPES OF COUNTING PROBLEMS

		Permutations (ordered)	Combinations (unordered)
بدون ارجاع	No repetition allowed	$\dfrac{n!}{(n-k)!}$	$\binom{n}{k} = \dfrac{n!}{k!\,(n-k)!}$
مع ارجاع	Repetition allowed	n^k	$\binom{n+k-1}{k}$

Example Calculate $\mathbf{P}(10, 3)$, the number of photographs of 10 friends taken 3 at a time.

$\mathbf{P}(10, 3) = 10 \cdot 9 \cdot 8 = 720.$
Note that you start with 10 and multiply 3 numbers.

A general formula, using the multiplication principle:

$$\boxed{\mathbf{P}(n, k) = n \cdot (n-1) \cdot (n-2) \cdots (n-k+1).}$$